Algebra Families

From a mathematical perspective!
An Introduction to algebra.

A unique learning experience

Student Learning Booklet

Version: 13/7/10

Prince Alfred College Founded 1869

Noel Baker Centre for School Mathematics

CASIO

Copyright Information.

The materials within, in their present form, can be used free of charge for the purpose of facilitating the learning of children in such a way that no monetary profit is made.

The materials within, in their present form, can be reprinted free of charge if being used for the purpose of facilitating the learning of children in such a way that no monetary profit is made.

The materials or ideas cannot be reproduced in any other publications without the express permission of the author.

Acknowledgements.

The 'approach' within is that developed by Anthony Harradine. The process of development has occurred over more years than we care to admit. The approached has been effected by a number of people; they are acknowledged below:

- The numerous teachers (in SA and NSW) from the schools who have tried this approach and found it to work.
- Alastair Lupton (Prince Alfred College, SA) for his thoughts on part of this work and for the addition of the 'generation' factor. Also for donating his Year 8 class to trial the most recent version of these materials (February 2005) at PAC and for his input during the most recent trial phase, specifically the notion of 'control'.
- Kate O'Donnell (MLC, Sydney) for her thinking (2003) about the delivery of the approach enclosed.
- Marty Schmude (St. Joseph's College, NSW) for being a guinea pig.
- Jan Wallace and the teachers from the schools involved in the CAS Pilot (NZ)

Pre-requisite knowledge.

This unit is a part of a 4-unit learning sequence, this being the first unit. It is assumed that students have mastered the fundamentals of operating with numbers – mainly positive and negative integers.

Using this resource.

This resource is ***not*** a text book and should not be used like one.

It contains material that is, in fact, expected to be covered as a rich dialogue between students and teacher and/or students and students.
You, as a teacher, must plan carefully 'your performance'. The inclusion of all the 'stuff' is to support:

- you (the teacher) in how to plan their performance – what questions to ask, when and so on,
- the student that may be absent
- parents or tutors who may be unfamiliar with the way in which this approach unfolds.

1. A little magic to being.

Do you believe in magic? Write down what happens here.

1.1 Teacher Magic

Choose an integer. Subtract one from the integer you chose, add one to the integer you chose. Now multiply these three integers together and then divide it by the integer you chose.

Now write down a list of ? (a number chosen by your teacher) consecutive integers, one of which is the number you ended with – do not tell the teacher which one it is though.

I bet they know what you started with.

2. A look at some human families (and some non-human families) and their features.

For each of the following families identify the features that set them apart from many other families.

Task 1

1) Suggest a ***surname*** that would suit this family.

2) Suggest a ***first name*** that would suit each ***individual*** member of this family.

Family One

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

Family Two

Family Three

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

Family Four

This poor little fellow has lost his family. What are his 'special' features? What might his surname be? What might his first name be?

<u>Summary</u>
So, in our world **families** are a **collection of individuals** that have a **surname** and a **first** name.

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

3. Representations of human families.

In the previous section we looked at a number of photographs. Each **photograph** was a **representation** of some of the members of each family.

1) Are there other ways of representing families? List as many as you can think of.

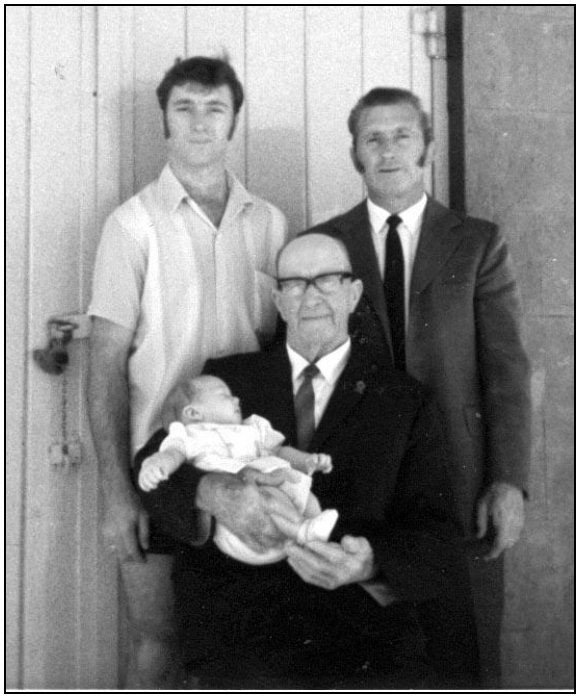
2) Does the representation include one, a few or all members of the family?

3) Bring in some different representations of your family and display them in your classroom.

Some place to display and document.

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

4. Generations



The concept of *generations* in human families can be displayed in a picture like the one seen above.

This picture shows four generations of one family – Great Grandfather, Grandfather, Father and child.

1. If you define yourself as being *generation zero*, List the people in your family in generations …,-3,-2,-1,0,1,2,3,…….

2. How many generations can you go back? When will you be able to move forward?

If you have a photo similar to the one above bring it (or a copy) into school so it can be displayed.

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

5. Non-human families.

The concept of family can be applied to many different situations. In geometry we often talk of *families of shapes*: the family of three sided plane figures, the family of convex polygons and so on.

By applying the concept of family to a variety of situations (including numbers) in a rather creative manner we can start to learn a great deal about the way mathematicians think about mathematics. Consider the following situations.

5.1 The family of divided lines.

Draw a straight line in the middle of an A-4 piece of paper that is 20 cm long. Label the end points P and Q.

Starting at P move your pencil some distance towards Q. When you have travelled far enough (or the distance you like, don't look at any one else), place a dot (point) on the line at that position. Label this point U.

Write your name on the paper and pin it on the wall with all your classmates divided lines.

These are all members of ***one*** family. How could we name each member (What is the same? What is different?)? Apply the concepts of:

- generation
- first name and
- surname.

Once you have agreed how to do this take down your line down and write on it, "This line belongs to XXX. It's name is _____ and it is the ____ generation of this family.

<u>A space for some important notes.</u>

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

5.2 A family of rectangles.

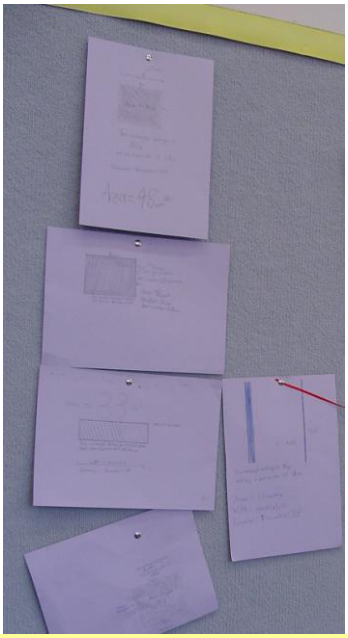
Make one rectangle with perimeter 28 cm. Do not look at what your classmates are doing. Make it the way you like.

Once each student has made one rectangle, pin them all on the wall.

These are all members of ***one*** family. How could we name each member (What is the same? What is different?)? Apply the concepts of:

- generation
- first name and
- surname.

Once you have agreed how to do this take down your rectangle and write on it, "This rectangle belongs to XXX. It's full name is _____ and it is the ____ generation of this family.



<u>A space for some important notes.</u>

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

5.3 A family of boxes.

Make a box from and A-4 piece of paper by cutting *squares* from the corners.

Once each student has made one box, pin them all on the wall near the rectangles family.

These are all members of ***one*** family. How could we name each member (What is the same? What is different?)? Apply the concepts of:

- generation
- first name and
- surname.

Once you have agreed how to do this take down your box and write on it, "This box belongs to XXX. It's name is _____ and it is the ____ generation of this family.

A space for some important notes.

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

5.4 The family of folded squares.

Your teacher will supply each student with a **8 cm square** of paper/light cardboard.

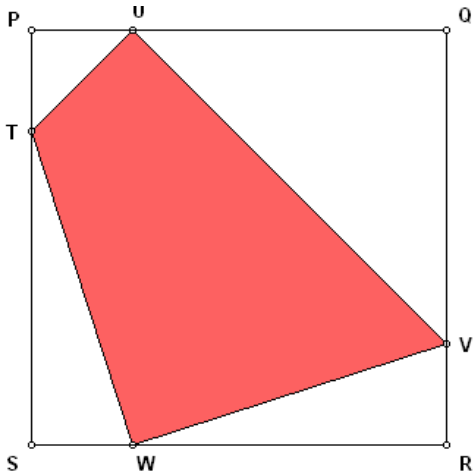
You are going to make four folds by following the following rules:

- Fold one (T to U) is made so that TP=PU.

You do not have to make this fold look like that seen opposite. PU could be small or large in length. Be an individual – do what you want, fold it at a length you 'like'.

- Fold two (U to V) is made so that UQ=QV.
- Fold three (T to W) is made so that SW=PU
- Fold four goes from W to V.

These are all members of ***one*** family.



Who made the square with the largest quadrilateral TUVW? Calculate the area of the quadrilateral TUVW that your folded square has.
Once done write the area you calculated on the white board.

Pin all of the folded squares on the wall near the rectangle family and the box family.

How many of these folds did you have **control** over?

<u>A space for some important notes.</u>

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

6. Something for you to practice – 1.

1) Consider the following family of numbers.

Number TINK multiplied by.		*Number TINK was thinking of.*	*Operator used by TINK (+ or -)*	*Number TINK added or subtracted.*		*The number TINK 'gets'.*
5	×	-4	-	2	=	-22
5	×	6	-	2	=	28
5	×	1	-	2	=	3
5	×	9	-	2	=	43
5	×	11	-	2	=	53
5	×	-6	-	2	=	-32
5	×	2	-	2	=	8

a) suggest a surname for this family
b) what generation is the individual with first name –32?
c) Who is generation 12?
d) Who generation is 2?
e) Supposing we can only have integer generations, is 57 a member of this family?

2) Consider the following family of numbers.

Number TINK multiplied by.		*Number TINK was thinking of.*	*Operator used by TINK (+ or -)*	*Number TINK added or subtracted.*		*The number TINK 'gets'.*
6	×	8	+	5	=	53
6	×	-2	+	5	=	-7
6	×	5	+	5	=	
6	×	11	+	5	=	
6	×	-7	+	5	=	-37
6	×	1	+	5	=	
6	×	2	+	5	=	17

a) suggest a surname for this family
b) Complete the table.
c) What generation is the individual with first name 53?
d) Who is generation 12?
e) Who generation is 2?
f) Supposing we can only have integer generations, is 59 a member of this family?
g) Note that 53 is a member of this family and the family from question 5. How does 53 differ in each family?

3) Find two families to which 28 belongs, and within which it is the same generation.

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

7. Correct mathematical terms.

One example of a TINK output can be seen below.

Number TINK multiplied by.		*Number TINK was thinking of.*	*Operator used by TINK (+ or -)*	*Number TINK added or subtracted.*		*The number TINK 'gets'.*
5	×	-4	-	2	=	-22
5	×	6	-	2	=	28
5	×	1	-	2	=	3
5	×	9	-	2	=	43
5	×	11	-	2	=	53
5	×	-6	-	2	=	-32
5	×	2	-	2	=	8

Things in common:

- multiplication by five
- subtraction of 2

Things different:

- the number being multiplied by five
- the result (or the number TINK gets)

We can say that this family has the *surname*

- Five times some number subtract two or
- $5 \times x - 2$ or
- $5x - 2$

where x is a letter that takes the place of the **infinitely many numbers** that YOU could have thought of. In the last section we learned we could think of this as the *generation* of the family member.

How many generations are possible?

When a letter takes the place of many different numbers it is called a ***pro-numeral***. (pro– meaning *substituting for*). Pro-numerals are normally presented in *italic* font. A pro-numeral may represent a physical quantity or simply just a number.

The letter chosen as a pro-numeral is often x, but can be anything the user likes. In many cases two or more different pro-numerals are required.

The branch of mathematics that deals with the study and use of pro-numerals is called ***Algebra***.

When a pro-numeral is written next to a number, like $5x$ it is assumed it means $5 \times x$ (just mathematicians being lazy!).

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

A collection of numbers, operations and pro-numerals, like $5x - 2$ is called an ***algebraic expression***. We learned in the last section we could think of this as the *surname* of the family. It describes **all** members of the family

An algebraic expression can have many **values** (family members). We learned in the last section we could think of this as the *first name* of the family member.

Can you see how an expression, like $5x - 2$, captures all of the members of the family? If not discuss this with others.

Make a poster, or use the space below to make a diagram that shows how the correct mathematical terms link to the words we used in the last section: surname, first name and generation

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

8. Something for you to practice – 2.

In the following exercises, perform all calculations *without* the use of a calculator.

1) Study the following number family.

5	×	3	-	4
5	×	-7	-	4
5	×	2	-	4
5	×	1	-	4
5	×	-4	-	4
5	×	11	-	4
5	×	7	-	4

a) Write down an algebraic expression that describes each member of this family.
b) Find the value of the expression when the pro-numeral you chose takes the value of 7.
c) Find the value of the expression when the pro-numeral you chose takes the value of 9.
d) Find the value of the expression when the pro-numeral you chose takes the value of –12.
e) Determine the value of the pro-numeral when the algebraic expression has value 19.
f) Determine the value of the pro-numeral when the algebraic expression has value 54.
g) Determine the value of the pro-numeral when the algebraic expression has value 25.
h) How many members are in this family?

2) Study the following number family.

9	×	3	+	7
9	×	-7	+	7
9	×	2	+	7
9	×	1	+	7
9	×	-4	+	7
9	×	11	+	7
9	×	7	+	7

a) Write down an algebraic expression that describes each member of this family.
b) Find the value of the expression when the pro-numeral you chose takes the value of 14.
c) Find the value of the expression when the pro-numeral you chose takes the value of 7.
d) Find the value of the expression when the pro-numeral you chose takes the value of –15.
e) Determine the value of the pro-numeral when the algebraic expression has value 115.
f) Determine the value of the pro-numeral when the algebraic expression has value 61.

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

g) Determine the value of the pro-numeral when the algebraic expression has value 26.
h) How many members are in this family?

3) Study the following number family.

13	×	3	+	7	+	9	×	3	-	2
13	×	-7	+	7	+	9	×	-7	-	2
13	×	2	+	7	+	9	×	2	-	2
13	×	1	+	7	+	9	×	1	-	2
13	×	-4	+	7	+	9	×	-4	-	2
13	×	11	+	7	+	9	×	11	-	2
13	×	7	+	7	+	9	×	7	-	2

a) Write down an algebraic expression that describes each member of this family.
b) Find the value of the expression when the pro-numeral you chose takes the value of 14.
c) Find the value of the expression when the pro-numeral you chose takes the value of 7.
d) Find the value of the expression when the pro-numeral you chose takes the value of –15.
e) Determine the value of the pro-numeral when the algebraic expression has value 115.
f) Determine the value of the pro-numeral when the algebraic expression has value 61.
g) Determine the value of the pro-numeral when the algebraic expression has value 26.
h) How many members are in this family?

4) Study the following number family.

9	-	4	×	1
9	-	4	×	2
9	-	4	×	3
9	-	4	×	4
9	-	4	×	5
9	-	4	×	6
9	-	4	×	7

Write down an algebraic expression that describes each member of this family.

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

5) Study the following number family.

5	-	3	×	11	×	11
5	-	3	×	-2	×	-2
5	-	3	×	3	×	3
5	-	3	×	7	×	7
5	-	3	×	15	×	15
5	-	3	×	-26	×	-26
5	-	3	×	7	×	7

Write down an algebraic expression that describes each member of this family.

6) Study the following number family.

2	×	11	×	11	+	5
2	×	-2	×	-2	+	5
2	×	31	×	31	+	5
2	×	-5	×	-5	+	5
2	×	1	×	1	+	5
2	×	6	×	6	+	5
2	×	17	×	17	+	5

Write down an algebraic expression that describes each member of this family.

7) Study the following number family.

2	×	11	×	1	+	5
2	×	-2	×	2	+	5
2	×	31	×	5	+	5
2	×	-5	×	4	+	5
2	×	1	×	-7	+	5
2	×	6	×	12	+	5
2	×	17	×	-14	+	5

Write down an algebraic expression that describes each member of this family.

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

9. Representing families of numbers – in a number of different ways.

Just as we can have multiple representations of human families (see section 3), we can also have *multiple representations* of the sorts of number families we have been working with.

Let us consider the following family.

Note that we have put some order into the *generations* we are looking at. It is common practice to do this.

2	×	1	-	3
2	×	2	-	3
2	×	3	-	3
2	×	4	-	3
2	×	5	-	3
2	×	6	-	3
2	×	7	-	3

Representation 1 – A structure table
You can see this above. This is not a very common way to represent a family, but a very important one for your understanding as it shows the *structure* of the family.

Representation 2 – An algebraic expression

$$2x - 3$$

Be sure you understand how this *captures* **all** the family members.

Representation 3 – A table of values
But what values? The essence of the family can be captured by table in which the pro-numeral value (generation) and the individual member's value (first name) are shown. Something like the following:

x	$2x$-3
1	-1
2	1
3	3
4	5
5	7
6	9
7	11
8	13

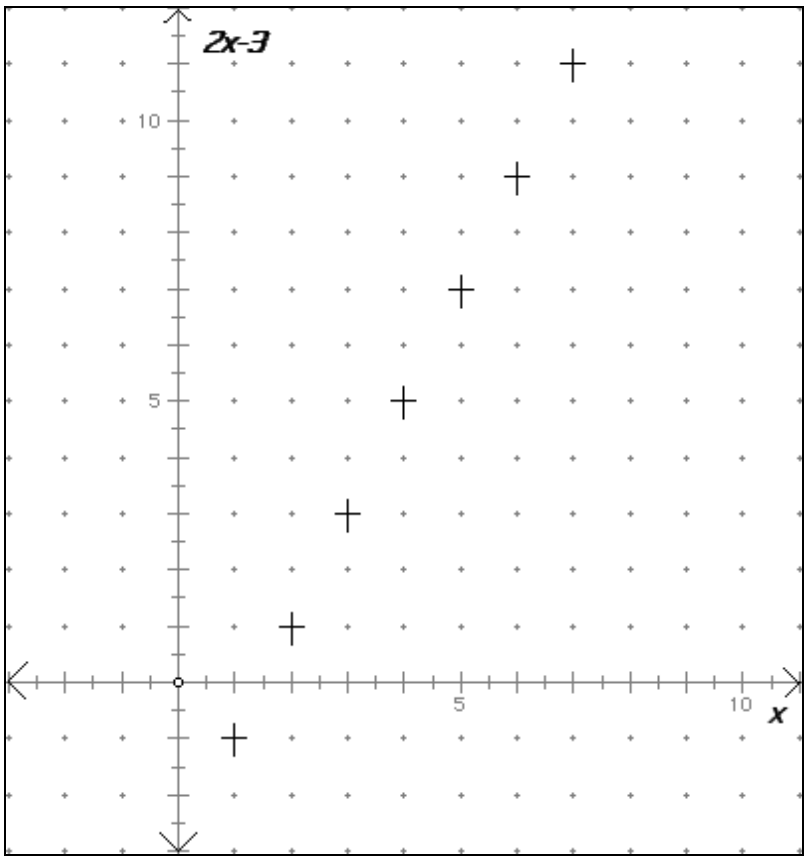
© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

Representation 4 – A graphical display – scatter plot **(or the Kodak happy snap).**

A family like this can also be represented on a graph where the horizontal axis is a measure of the generation (x) and the vertical axis is a measure of the value of the expression using that particular pro-numeral.

The value of the family member is normally assigned the vertical axis as its value might go 'up and down' and hence it is seen to do just that on the graph. The generation concept fits the left to right idea of the graphs horizontal axis. You will see families that go up and down in value as generations pass a little later.

Note that both axes are incremented in the same way – we call this a *square* set up. You will see later that a square set up is important in many cases. The Kodak snap seen earlier on the CP 300 may not have been square.



© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

10. Something for you to practice – 3.

Provide three different representations for each of the following families (**in addition to the one that is given**).

a)

-5	×	1	+	3
-5	×	2	+	3
-5	×	3	+	3
-5	×	4	+	3
-5	×	5	+	3
-5	×	6	+	3
-5	×	7	+	3

b)

5	×	-3	+	4
5	×	-2	+	4
5	×	-1	+	4
5	×	0	+	4
5	×	1	+	4
5	×	2	+	4
5	×	3	+	4

c) $4x - 1$ for integer values of x from –4 to 4.

d) $5 - 3x$ for integer values of x from –4 to 4.

e)

x	??
1	-3
2	0
3	3
4	6
5	9
6	12
7	15
8	18

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

11. Introducing a second pro-numeral.

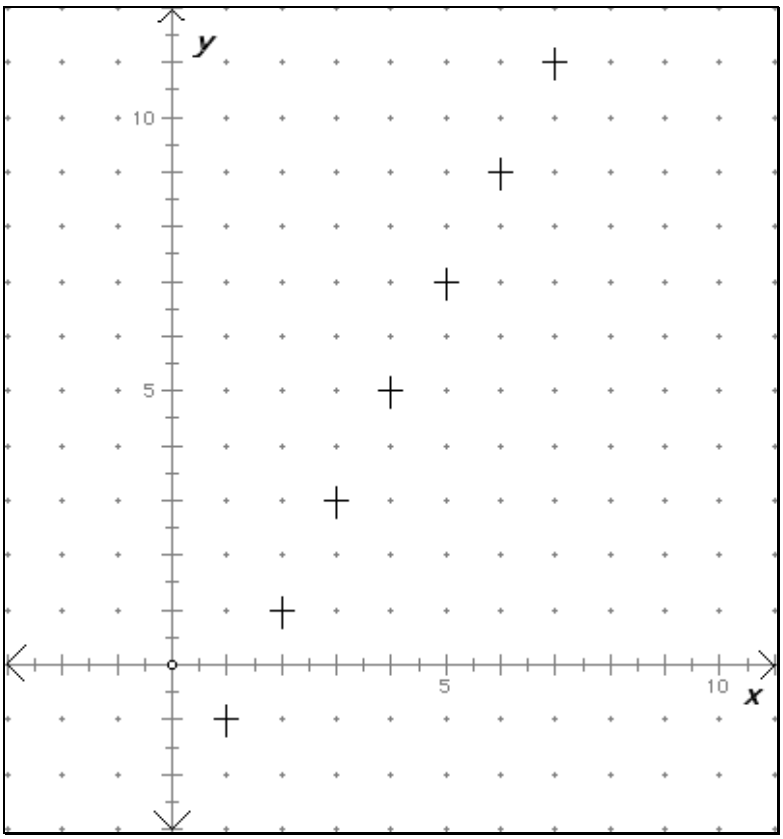
Often, we introduce a **second pro-numeral** to take the place of the value of the family member (first name). In doing this we introduce something called an algebraic equation (because it includes and equal sign). It looks as follows:

$$y = 2x - 3$$

and so the table of values looks as follows (no real difference):

x	y
1	-1
2	1
3	3
4	5
5	7
6	9
7	11
8	13

The graph also has only one small change:



© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

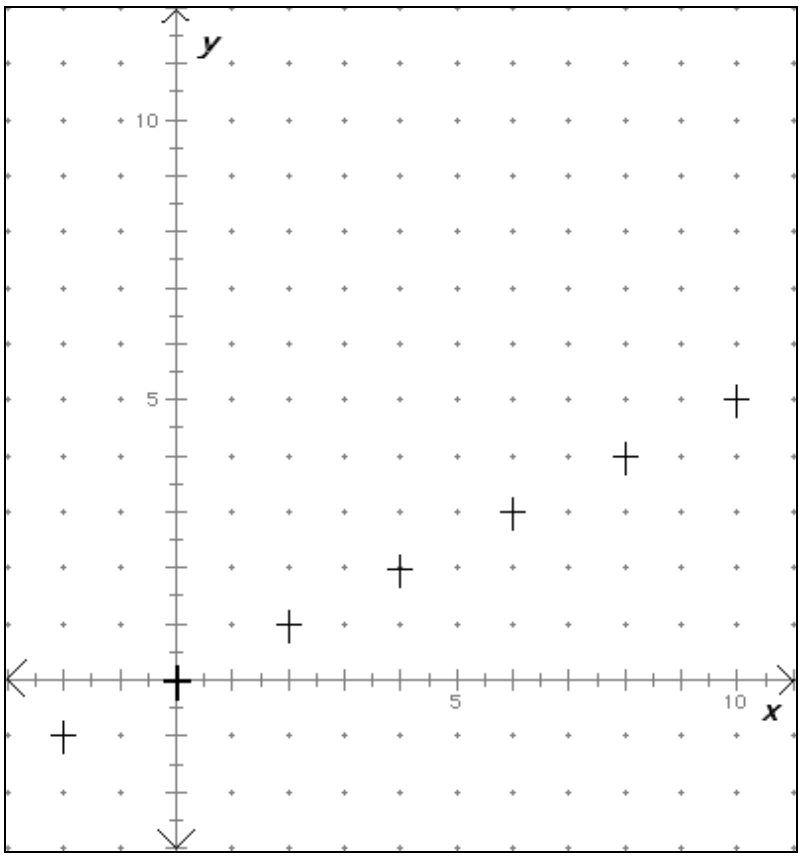
12. Something for you to practice – 4.

1) Provide three different representations for the family known as $y = -3x + 5$

2) Provide three different representations for the family known as $y = 2x - 3$

3) Provide three different representations for the family known as $y = 6 - 2x$

4) Provide three different representations for the family representing the following graph.



© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

13. Something for you to practice - 5

1. Go back to section 15 and produce displays for each family in that section using your CP 300. Check that the displays agree with your 'paper and pen' work. Be sure to set up the view window as you drew your axes.

2. For each of the following families, make a table of values, and a scatter plot representation.

   Display both positive and negative generations.

   View both square and non-square scatter plots of each family.

   a) $y = 25 - x$
   b) $y = 10 - x$
   c) $y = 10 - x^2$
   d) $y = x^2 - 10$

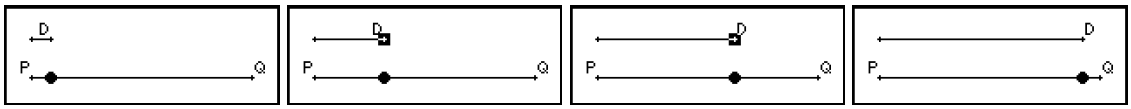
© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

14. Capturing ALL the individuals of a family in one just one diagram.

Who made the folded square with the quadrilateral of biggest area? Did you ***prove beyond any doubt*** your result? Believe it or not, one way to prove your result beyond any doubt is to capture all family members with one still diagram and then work on that.

14.1 The family of divided lines – re-visited.

Let's start with the family of divided lines:



The *control stick* allows us to *control the length of P to the dividing point*. It allows us to display a variety of generations of family.

Answer the following:
Let the dividing point be R. If PR is 1 unit, how long is RQ?
Let the dividing point be R. If PR is 1.2 units, how long is RQ?
Let the dividing point be R. If PR is 3 units, how long is RQ?
Let the dividing point be R. If PR is 4.6 units, how long is RQ?
Let the dividing point be R. If PR is 7.9 units, how long is RQ?

One way we can capture this in one static diagram is to give the **length** *P to the dividing point* a name that captures the fact it can be infinitely many values. We could do this as follows:

- Let the dividing point be R
- Let the infinitely many lengths of PR be represented by x.

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

Hence, we could represent this family with a diagram like that which follows:

So, how can we represent the infinitely many lengths of RQ with one simple algebraic expression? Think about it – discuss it, give reasons why what you think might work, works.

Put your representation on the diagram below.

Now, write down an algebraic expression for the length of PQ
But how long is PQ – always?

Some room for doing some work.

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

15. Displaying all the members of the family - graphically.

Earlier in this unit we dealt with situations where we only considered ‘integer generations’.

To describe the situations we have seen in the animations we clearly need to consider ‘in between generations’ or non-integer generations. This is also true of some of the earlier situations we looked at.

15.1 The family of divided lines re-visited

Consider the simple animation we worked with in section 18.1 – the family of divided lines.

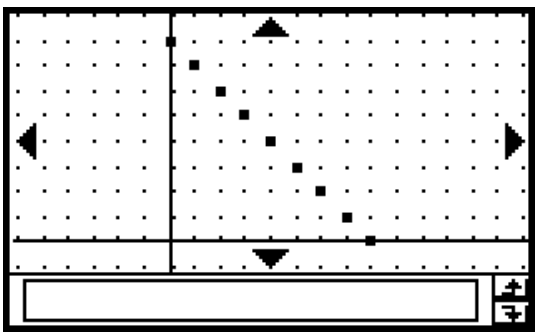
Suppose we what to display the length of RQ for each of the possible lengths of PR.

Let L be the length of RQ. So, $L = 8 - x$

There is clearly infinitely many lengths of PR as R can be in infinitely many positions. So a table could be constructed as follows:

x	L
0	8
1	7
2	6
3	5
4	4
etc	

A corresponding graph would look as follows:



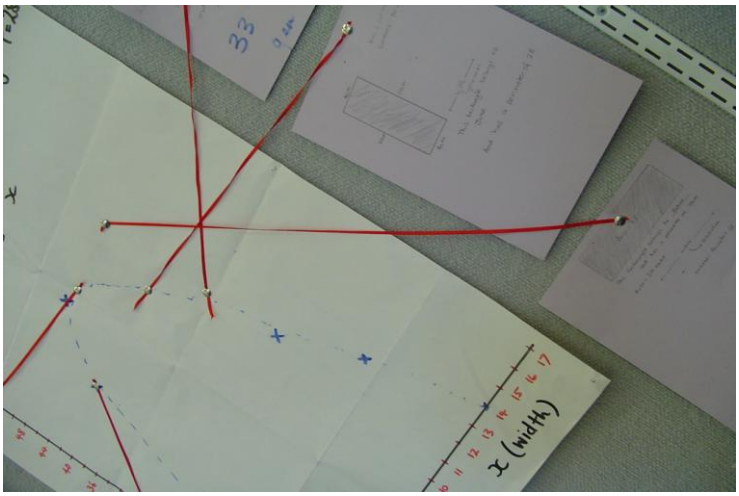
© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

x	L
0	8
0.01	7.99
0.02	7.98
0.03	7.97
etc	

This would be a very long table (and we could make it longer). What would the corresponding graph look like?

16. Time for a very big graph.

Join in with the rest of your classmates as you draw a very large graph to assist your understanding of the representations possible for the family of rectangles that you have made.

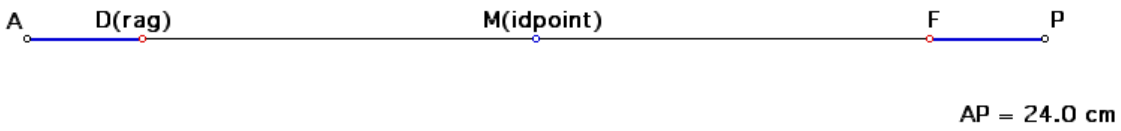


Return to Section 20 if you did not meet the challenges earlier.

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

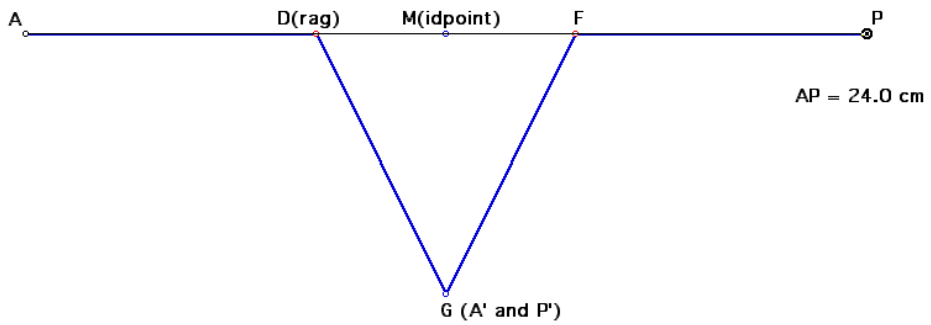
17. Something to practice – 8.

1. The line AP, below, represents a line 24 cm long.
   M is the mid-point of the line.
   D is a point that can be anywhere on the segment AM.
   Length DM is always the same as MF.



   Add to the diagram so that it represents all the possible positions of D.

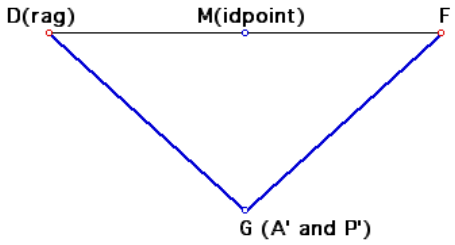
2. Consider the following diagram that represents a family of triangles DFG. It is build from the line in Question 1.
   DG is always the same length as AD and FG is always the same length as FP



   a) How long is DF (always)?
   b) How long is DM (always)?
   c) How long is MF (always)?
   d) How long is DG (always)?
   e) How long is GF (always)?
   f) What is the length of DM when the triangle is no longer a triangle?
   g) **How** would you find an algebraic expression that represents the **area** of every triangle in this family?

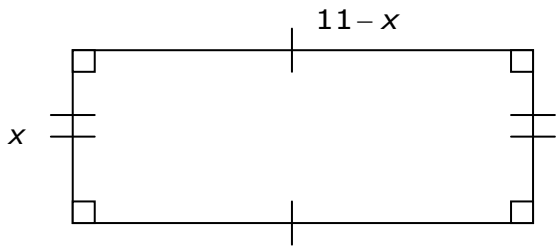
© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

3. Consider the diagram below that represents all triangles in a particular family of **isosceles** triangles.



a) If DM is said to be x units long how long is MF (always)?
b) If DG is (27-x) units long, how long is GF (always)?
c) What is the perimeter of every triangle in this family? Explain how you arrived at your answer?
d) Draw a graph of perimeter vs x.

4. a) What is the perimeter of all the rectangles represented below?



b) Write down an algebraic expression that represents the area of all of the possible rectangles.
c) Draw a graph of Area vs x marking on the graph a representation of 6 of the rectangles that belong to this family

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

18. Something for you to try – 2.

1. Write down **all** pairs of consecutive integers.

2. a) Write down the first 10 consecutive odd positive integers

   b) Write down 1 algebraic expression that summarises all of these 10 and all other odd positive integers that exist.

3. a) Write down the first 10 consecutive even positive integers

   b) Write down 1 algebraic expression that summarises all of these 10 and all other even positive integers that exist.

4. Write down **every** 2-digit number, and then every 3 digit number – in less than 30 seconds.

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

19. Using simple algebra to do powerful things – proof, beyond any doubt.

It is hoped that you have a strong feel that algebra (the use of pro-numerals) is all about writing, in a very simple manner, a statement that covers an infinite number of individual cases for some situation.

In fact, algebra is about making what seems impossible, (working with the infinite), possible.

Consider the following:

> Each student in the class is to choose **two consecutive** numbers and then add them together. Compare your results, what do they all have in common? Make a conjecture about the sum of **every pair** of consecutive numbers. Can you prove this conjecture to be true – beyond any doubt?

Some room for some important notes.

1. Prove that the sum of any two consecutive odd numbers is an even number (try a few individual cases first to check it out.

2. Prove that the sum of any two consecutive even numbers is an even number (try a few individual cases first to check it out.

3. Prove that the sum of a multiple of 5 and the 'equivalent generation' multiple of 3 is a multiple of 8, but that it is not so for the sum of any multiple and 5 and multiple of 3.

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

20. Back to the magic - 1.

20.1 Teacher magic – re-visited.

Remember this:

Choose an integer. Subtract one from the integer you chose, add one to the integer you chose. Now multiply these three integers together and then divide it by the integer you chose. Now write down a list of ? (a number chosen by your teacher) consecutive integers, one of which is the number you ended with – do not tell the teacher which one it is though. I bet they know what you started with.

Can you now work out how the teacher knew? Try to make an algebraic expression

Some room for some work.

26.2 Some slightly simpler magic.

* Choose a Number
* Add 5
* Double the result
* Subtract 4
* Divide the result by 2
* Subtract the number you started with
* Is the result is 3?

Pick 4 different starting numbers and show that the result is 3 in each case. Does this prove the result will be three no matter what the starting number – i.e. in every possible case? Can you prove it?
Try to make an algebraic expression

Some room for some work.

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)

20.3 Yet more magic.

Experiment with the following bits of 'magic'. Prove that the resulting number is always the same

Magic 1
Choose a number. Add 3. Multiply by 2. Add 4. Divide by 2. Subtract the number you started with. The result is ??.

Magic2
Choose a number. Double it. Add 9. Add the number you started with. Divide by 3. Add 4. Subtract the number you started with. The result is ??.

Some room for some work.

20.4 Creating your own magic.

Create a trick of your own. You must prove, using algebra, that your trick will always work – would hate to let the world down!

© Anthony Harradine and Garry Turner, 2003 - 2010, all rights reserved (WIP)