Karate chop

The physics of tameshiwari

by A. Biryukov

TAMESHIWARI IS A KARATE term that means the testing of one's psychological training and of the skill to strike various objects with the hand. Karate came to the western world from Okinawa, Japan. It was developed in the sixteenth and seventeenth centuries, when in fear of rebellions, the governing powers confiscated all weapons from the people, including their ritual and kitchen knives. It was beyond the power of the peasants to fight the armed-to-the-teeth Samurais with bare hands, but they could repel a gang of bandits using karate.

Perhaps this explains the origin of tameshiwari, which is always interesting for spectators and produces the impression of a miracle upon the uninitiated. Today the skill of tameshiwari is most often shown in demonstrations and competitions of karate, where the targets are planks of certain sizes made of coniferous (soft) wood.

We consider in this article a simple physical model of a hand hitting a plank, which yields some estimates and advice, and evaluates the possible limits of athletic achievements in tameshiwari. To find a number of parameters for this model, we must solve several preliminary problems, which are interesting in themselves. However, to keep our train of thought running on the main line, we solve these problems in the appendixes at the end of the article.

Let a blow be struck with a fist of mass m arriving with speed v at the center of a plank of dimensions d, l, and h that lies on two supports (fig. 1). The fibers of the wood are parallel to the supports, which are separated by approximately the length of the plank l. One of the "secrets" of karate says that to enhance the effectiveness of the blow, one should apply force F to the accelerated fist just before the moment of contact and maintain it during the entire collision. We consider the deformation of the plank in the reference system shown in fig. 2. Let x_0 be the displacement of the plank's center from its equilibrium position. Assume that the breaking of the plank (signaled by the breaking of its surface) occurs at some critical value $x_0 = x_r$, when the stress σ (the force applied to a unit area of the plank's cross-section) at the plank's surface reaches some critical value σ_r, which depends on the strength of the material.

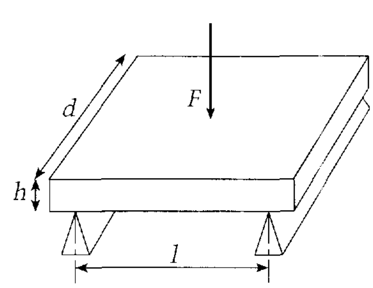


Figure 1

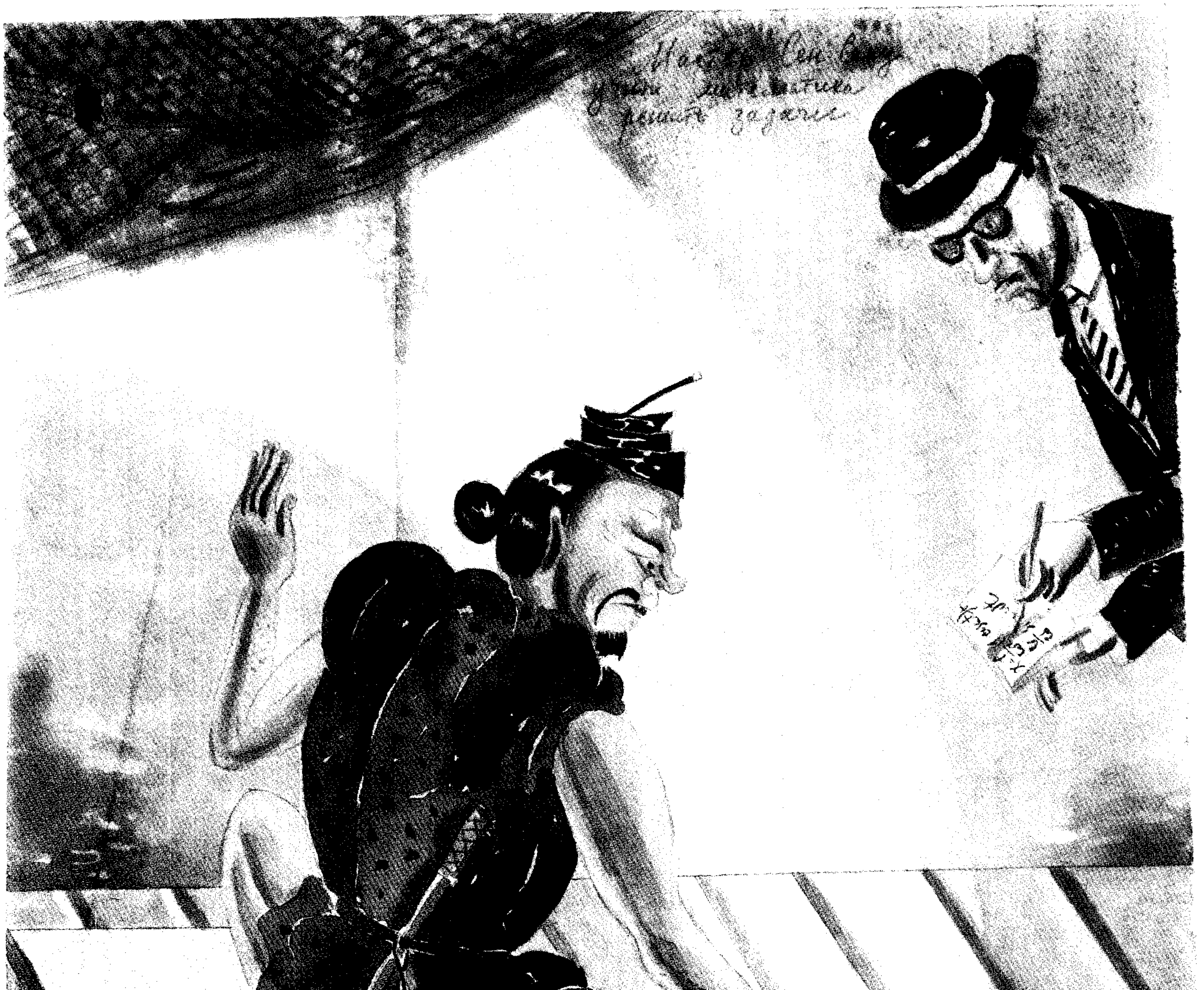
First we find the relationship between x_r and σ_r, which is determined by the elastic properties and geometry of the plank. The maximum bending and the maximum stress at the surface of the plank will take place at its center. In Appendix 1 we show that this stress is given by the formula

$$\sigma = \frac{Yh}{2R},$$

where R is the radius of curvature of the central line CC in the middle of the plank (fig. 2) and Y is Young's modulus for the type of wood.

Now we assume a particular shape for the deformed plank and take into consideration that its ends are fixed at the points $y = \pm l/2$, and the maximum displacement from equilibrium occurs at the center of the plank. Note that the exact shape

Art by Leonid Tishkov



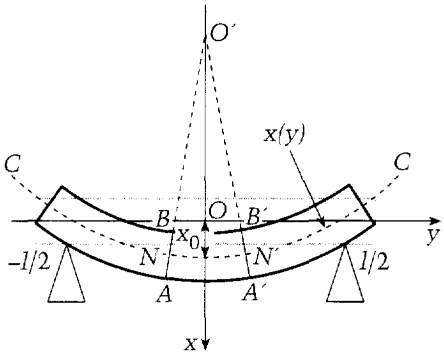


Figure 2

of the plank depends on the specific (and quite clearly understood) conditions of the interaction between the contact surface of the fist with the plank (in a correct blow, contact is made with the knuckles of the middle and index fingers). In our calculations we will use a practical formula based on experimental data, which makes it possible to obtain simple estimates.

Let's approximate the bending of the plank by a cosine between the points $y = \pm l/2$. In this case, the displacement x of any point along the central line depends on its coordinate y as

$$x(y) = x_0 \cos\left(\frac{\pi y}{l}\right).$$

Appendix 2 shows that the corresponding radius of curvature at the plank's center will be

$$R = \left(\frac{l}{\pi}\right)^2 \frac{1}{x_0}.$$

Plugging this into the formula for σ yields the stress on the surface of the plank at its middle when the plank's center is displaced by x_0:

$$\sigma = \frac{x_0 Y h \pi^2}{2l^2}.$$

This formula shows that breaking ($\sigma = \sigma_r$) occurs when the plank's center is shifted by

$$x_r = \frac{2\sigma_r l^2}{\pi^2 Y h}.$$

Now we model the elastic properties of the plank with a spring with a spring constant k, which is loaded by an external force. This spring constant is found in Appendix 3 to be

$$k = \frac{\pi^2 Y h^3 d}{3l^3}$$

Having determined the necessary parameters, we return to the initial dynamic problem of a fist hitting a plank. The motion of the fist is described by Newton's second law:

$$mx'' = -kx + F,$$

where x henceforth means the displacement of the fist from the initial contact position with the plank, and the primes indicate differentiation with respect to time.

To simplify, we consider the force F, which is applied to the fist by the arm, to be constant. Substitutions yield the following solution:

$$x = A\cos\omega t + B\sin\omega t + \frac{F}{k},$$

which includes two arbitrary constants A and B. To find them, we specify the initial conditions: $x = 0$ and $x' = v$ at $t = 0$. Now we get

$$x = \frac{f}{\omega^2}(1 - \cos\omega t) + \frac{v}{\omega}\sin\omega t,$$

where $f = F/m$ has dimensions of acceleration, and $\omega = \sqrt{k/m}$ is the frequency of natural oscillation of the fist under the action of the elastic force of the plank.

The next step is to find the maximum displacement x_{max} of the fist for the given initial speed v and force F. By equating the time derivative of x to zero with the subsequent elimination of t, we get

$$x_{max} = \frac{f}{\omega^2}\left(1 + \sqrt{1 + \left(\frac{v\omega}{f}\right)^2}\right).$$

To obtain the conditions of breaking, this displacement must be set equal to x_r, which yields the equation

$$\frac{2\sigma_r h^2 d}{3Fl} = 1 + \sqrt{1 + \frac{\pi^2 Y h^3 v^2 dm}{3F^2 l^3}},$$

connecting the properties of wood and the geometry of the plank with the parameters of the collision.

We solve this equation for F, again using the parameters x_r and k:

$$F = \frac{kx_r}{2} - \frac{mv^2}{2x_r}.$$

For the plank to break, this force must be applied at the moment of contact to the fist moving with initial speed v. We can see that if the speed of the fist is large enough, the value of F becomes negative, so no force is needed to break the plank with a moving fist (in a similar way we need not apply force to a hammer when driving a small nail into wood). In this case the initial speed of the fist must be larger than

$$v = x_r\omega = \frac{2\sigma_r}{\pi\sqrt{3}}\sqrt{\frac{lhd}{mY}},$$

which is proportional to the square root of the plank's thickness h. By contrast, if the initial speed of the fist is zero, then this formula shows that to break the plank, the force must be no less than

$$F = \frac{kx_r}{2} = \frac{h^2\sigma_r d}{3l},$$

which is proportional to the square of the plank's thickness h. Therefore, to break a thicker plank, it is more practical to increase the speed of the blow, not its force.

Now let's solve the equation that determines the condition of the plank's breaking relative to its thickness h. It yields the thickness of the plank that can be broken for the given parameters of the blow:

$$h = \frac{3\pi^2 Y v^2 m}{8\sigma_r^2 l d}\left(1 + \sqrt{1 + \frac{64 F l^3 \sigma_r^3 d}{3\pi^4 Y^2 v^4 m^2}}\right).$$

Let's obtain some estimates, using the following experimental parameters for the wood: $E = 10^8$ N/m^2 and $\sigma_r = 5 \cdot 10^6$ N/m^2. The standard plank in tameshiwari has a width of 20 cm and a length of 30 cm. We assume $l = 25$ cm, because the ends of

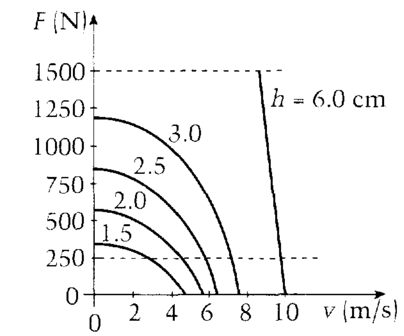


Figure 3

the plank (located beyond the supports) can be neglected. The mass of the fist is assumed to be 1 kg, and this number takes into account the forearm as well. Figure 3 shows the dependence of the force F on the initial speed v for various thicknesses h of the plank. If the combination of F and v corresponds to a point lying above the curve for a specified value of h, the plank will break.

Now we can evaluate the thickness of the plank that can be broken by a man. The force developed by the hand of a typical man is $F = 250$ N. Figure 3 shows that at $v = 0$ a typical man cannot break even a rather thin plank with a thickness of only 1.5 cm. To perform this deed, he must apply a force of about 300 N.

The experimental value for the maximum speed of the fist is about 10 m/s. Plugging $v = 10$ m/s and $F = 250$ N into the formula for h, we get the thickness of the plank: $h = 6$ cm. This value is rather large, and perhaps only experienced karate masters with excellent striking technique and psychological training can break such a thick plank. However, inquisitive readers can try to break a plank with a thickness of 2 cm, because the necessary values of force and speed can be achieved by the average person. In this process it is very important to follow the basic psychological "secret" of karate: Never doubt yourself.

Appendix 1

Let's find the stress on the surface of the plank. We consider two symmetrical cross-sections AB and $A'B'$ (fig. 2), which are normal to the line CC and separated by a small distance l_0 along this line. Consider the element $AA'B'B$. Due to its small value, we can approximate the curves AA', NN', and BB' by arcs with centers lying on the so-called axis of bending O', which is perpendicular to the page. The outer surface of the plank between points A and A' is stretched, while the inner surface between points B and B' is compressed. When bending is absent, the lengths of curves AA' and BB' are the same and equal to l_0 (the length of the central curve NN'), which retains its length during bending. Let R be the radius of curvature of the line NN'. Then $l_0 = R\alpha$, where α is the central angle subtending arc NN'. When the plank is not very thick—that is, when $h << R$, the length of curve AA' will be $l_1 = (R + h/2)\alpha$, and its elongation due to bending will be $\Delta l = l_1 - l_0 = h\alpha/2$. According to Hooke's law, the stress in the outer surface of the plank is

$$\sigma = Y\frac{\Delta l}{l_0} = \frac{Yh}{2R}.$$

Appendix 2

Let's find the radius of curvature of the surface of a bent plank at the middle point ($y = 0$). Recall that if R is the radius of curvature of any curve at a specified point, then the circle of radius R that passes through this point and whose center lies on the perpendicular to the curve at this point coincides (according to the definition of the radius of curvature) with the curve within a small distance of this point. When $|\pi y/l| <<$ 1, the function $x(y)$ becomes

$$x(y) = x_0 - \frac{x_0}{2}\left(\frac{\pi}{l}\right)^2 y^2.$$

Here we used the well-known approximation $\cos\gamma = 1 - \gamma/2$ for $|\gamma| << 1$.

The circle of radius R and center O' (fig. 2), which passes through the point $(x_0, 0)$ and which was considered in Appendix 1, is described by the equation

$$y^2 + (x - x_0 + R)^2 = R^2,$$

which can be easily solved to find the displacement $x(y)$:

$$x(y) = x_0 - R + R\sqrt{1 - \left(\frac{y}{R}\right)^2}.$$

Using another approximate formula, $\sqrt{1-\gamma} \cong 1 - \gamma/2$ for $|\gamma| << 1$, we get the following formula, which is correct for $|y/R| << 1$:

$$x(y) = x_0 - \frac{y^2}{2R}.$$

By comparing the two formulas for $x(y)$, we get the radius of curvature:

$$R = \left(\frac{l}{\pi}\right)^2 \frac{1}{x_0}.$$

Appendix 3

Let's find the dependence of the displacement x_0 of the center of a plank resting on two supports upon the external force F, which is distributed along the central fibers and directed downward. The mass of the plank will be neglected.

Due to the assumed symmetry, the force F is evenly distributed between the supports. We look at the cross-section through the plank at the plank's center (fig. 2) and consider the equilibrium condition for the left half of the plank. It is affected on the right by the external force $F/2$, which is applied near the edge and directed downward. This force is counterbalanced by the normal force of the left support. We can see that the sum of the torques relative to the plank's center will be determined only by the torque due to the left support:

$$\tau = \frac{Fl}{4}.$$

On the other hand, this torque is counterbalanced by the torques due to the tension and compression applied by the plank's right half on its left half in the plane of the cross-section. This torque can be derived from the formula for σ by modifying it to calculate the stress in the bulk of the plank along the y-axis. As follows from the derivation of this formula (Appendix 1), we must replace the displacement $h/2$ from line NN' corresponding to the point on the outer surface of the plank with the distance δ from this line $(-h/2 < \delta < h/2)$. In this case the stress in the bulk of plank will be

$$\sigma = \frac{Y\delta}{R}.$$

The total torque due to the elastic tension and compression forces relative to the plank's center will thus be equal to

$$\tau = \int_{-h/2}^{h/2} \delta\sigma d \cdot d\delta = \frac{Y}{R} d \int_{-h/2}^{h/2} \delta^2 d\delta = \frac{Yh^3 d}{12R}.$$

By plugging the value for the radius of curvature into this equation and equating the right-hand terms of the two formulas for τ, we get the relationship between the force F and the displacement x_0:

$$x_0 = \frac{3Fl^3}{\pi^2 Yh^3 d}.$$

This formula can be rewritten in the form $F = kx_0$, from which the formula for the spring constant k of the equivalent spring immediately follows:

$$k = \frac{\pi^2 Yh^3 d}{3l^3}.$$

Quantum on deformation and strength:

A. Borovoy, "Holding up under pressure," January 1990, pp. 30–32.

A. A. Dozorov, "Taking a flying leap," September/October 1990, pp.10–11.

A. Mineyev, "Trees worthy of Paul Bunyan," January/February 1994, pp. 4–10.

A. Leonovich, Kaleidoscope, "All bent out of shape," September/October 1995, pp. 32–33.

L. Grodko, "Wave on a car tire," November/December 1998, pp. 10–12.

How to be a QUANTUM author

Have you written an article that you think belongs in *Quantum*? Do you have an unusual topic that students would find fun and challenging? Do you know of anyone who would make a great *Quantum* author? Write to us and we'll send you the editorial guidelines for prospective *Quantum* contributors. Scientists and teachers in any country are invited to submit material, but it must be written in colloquial English and at a level appropriate for *Quantum*'s target readership of high school and college students. Send inquiries to:

Managing Editor
Quantum
1840 Wilson Boulevard
Arlington VA 22201-3000